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**AXISYMMETRIC STABILITY OF TRIPLET-SHAPED  
TOKAMAK PLASMAS**

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# AXISYMMETRIC STABILITY OF TRIPLET-SHAPED TOKAMAK PLASMAS

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Abstract Ideal MHD growth rates of axisymmetric modes of racetrack and triplet-shaped tokamak plasmas in rectangular conducting shells are computed. Assuming fixed  $q$ -values on axis and on the plasma boundary, it is found that the triplet is considerably more stable against axisymmetric modes than the racetrack.

## 1. INTRODUCTION

Current interest in elongated tokamaks (HELTON et al., 1986; REBUT et al., 1986; KÖEPPENDÖRFER, 1986; LUXON et al., 1986; OKABAYASHI et al., 1986; MARCUS et al., 1985) springs mainly from the hope to achieve higher beta values than are possible in conventional tokamaks with circular cross section. One of the major disadvantages of elongated tokamaks is the fact that they must rely on a conducting shell (or a set of passive coils mounted inside the vacuum vessel) for stabilizing axisym-

metric modes on the Alfvén time scale. The maximum allowable distance between the shell and the plasma boundary is typically between 15% and 25% of the horizontal minor radius (HOFMANN et al., 1985). This causes a number of problems. It will be difficult, for example, to install RF antennae with good coupling efficiency in the narrow space between plasma and wall. Also, there is a high risk of plasma contamination by impurity influx from the wall under these conditions.

In view of these problems, we have studied the question whether it is possible to improve the axisymmetric stability of elongated tokamaks by varying the plasma shape, while keeping the shape of the conducting shell fixed, and without reducing the minimum plasma-wall distance. In this Letter, we show that this is indeed possible, e.g. by transforming a racetrack-shaped plasma into a triplet.

## 2. EQUILIBRIA

Fig. 1 shows the basic configuration. The plasma is enclosed in a perfectly conducting shell with rectangular cross section. We assume that the three lobes of the plasma have the same width ( $2a$ ), and that the plasma-wall distance,  $\Delta$ , measured at the points of closest approach, is the same on all sides. This assumption is based on the observation that, for a given minimum plasma-wall distance, the centered plasma has the best stability properties. Moving either the inner or the outer wall away from the plasma will make the configuration less stable. A typical sequence of equilibria of this type is shown in Fig. 2. The indentation parameter,  $(a-c)/a$ , varies between 0 and 0.2. The source functions, which were used to generate these equilibria are given below:

$$p' = C_p [\alpha_p + \gamma_p x^n - (\alpha_p + \gamma_p) x^{n+1}] \quad (1)$$

$$T T' = C_T \mu_O R_O^2 \left[ 1 - \alpha_T x - \frac{\gamma_T}{m+1} x^{m+1} + \frac{\alpha_T + \gamma_T}{m+2} x^{m+2} \right]$$

where  $\gamma_p = (n+1)[n+2 - \alpha_p(n+1)]$

$$\gamma_T = (m+1)[m+2 - \alpha_T(m+1)]$$

and  $x = \frac{\psi_{ax} - \psi}{\psi_{ax} - \psi_{lim}}$

These functions satisfy the usual boundary conditions on the edge,  $p' = T T' = 0$  for  $\psi = \psi_{lim}$ . The resulting pressure and current profiles have similar shape, at low  $\beta$ , as can be seen in Fig. 2. The free parameters which appear in the source functions (1) were chosen in such a way as to keep the  $q$  values on the axis and on the edge constant ( $q_0 = 1.05$ ,  $q_S = 2.05$ ). This choice of  $q$ -values is consistent with a necessary condition for  $n = 1$  kink stability, i.e.  $q_0 > 1$ ,  $q_S > 2$  (TROYON et al., 1986). For the race-track equilibrium (Fig. 2a), the  $q$ -profile is chosen as flat as possible in the central region in order to obtain the widest possible current profile and thus maximum axisymmetric stability.

A summary of equilibrium parameters, corresponding to the three configurations shown in Fig. 2, is given in the Table, below:

Indentation, $(a-c)/a$	0	0.1	0.2
$C_p/C_T$	0.1	0.1	0.1
$\alpha_p = \alpha_T$	0.9072	0.5860	0.2000
$n = m$	3.0	3.0	3.0
plasma current, $\mu_0 I_p/R_0 B$	1.89	1.69	1.59
poloidal beta			
$\beta_p = (8\pi \int p ds)/(\mu_0 I_p^2)$	0.124	0.108	0.093
Aspect Ratio $R/a$	4.2	4.2	4.2
Elongation $b/a$	3.6	3.6	3.6

Current, pressure and  $q$ -profiles are shown in Fig. 2. It is seen that, with growing indentation, the  $q$ -profile develops an intermediate maximum, as the configuration approaches the creation of an internal separatrix.

### 3. AXISYMMETRIC STABILITY

We compute the ideal MHD growth rate of the most unstable axisymmetric mode of triplet-shaped plasmas, using the FBT code (HOFMANN et al., 1986). Calculations are performed for various indentations  $((a-c)/a)$  and plasma-wall distances  $(\Delta)$ . From these results we determine, for each equilibrium, the critical plasma-wall distance  $\Delta_c$  which leads to a marginally stable configuration. Fig. 3 shows  $\Delta_c$  as

a function of the indentation,  $(a-c)/a$ . We note that the stability improves as the indentation grows, inspite of the fact that the average plasma-wall distance increases. This surprising result can be explained when one considers the change in the plasma current distribution which is produced by the shape modification. Fig. 2 suggests that both the horizontal and the vertical current profiles become wider with increasing indentation. The gain in axisymmetric stability due to this profile modification is more important than the loss of stability caused by the increasing (average) plasma-wall distance. The result is a net gain in stability with increasing indentation.

#### 4. DISCUSSION

Figure 2 shows that triplet-shaped plasmas develop rather unusual  $q$ -profiles at high indentation. The effect of these profiles on non-axisymmetric stability is presently being investigated, using the ERATO code (GRUBER et al., 1981). Preliminary results (TROYON et al., 1986) show that the  $n = 1$  stability properties of triplets are generally similar to those of racetracks and D's, inspite of non-monotonous  $q$ -profiles.

From an experimental point of view, the triplet is obviously more difficult to create than the racetrack. If the shaping coils are placed relatively far from the plasma boundary, the currents in these coils must be larger for a triplet than for a racetrack. Whether this problem is serious or not, when compared with the gain in axisymmetric stability, remains an open question.

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FIGURE CAPTIONS

Fig. 1: Triplet-shaped equilibrium configuration with conducting shell.

Fig. 2: Racetrack and triplet-shaped equilibria with increasing indentation. a) Racetrack, b) and c) Triplets with  $(a-c)/a = 0.1$  and  $0.2$ , respectively. Horizontal current, pressure and  $q$ -profiles, taken at the height of the magnetic axis, are shown on the top. Vertical current profiles through the magnetic axis are shown on the right-hand side of each equilibrium. The plasma boundary is indicated by a dotted line.

Fig. 3: Plasma-wall distance  $\Delta_c$  for marginal stability as a function of indentation.

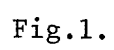


Fig.1.

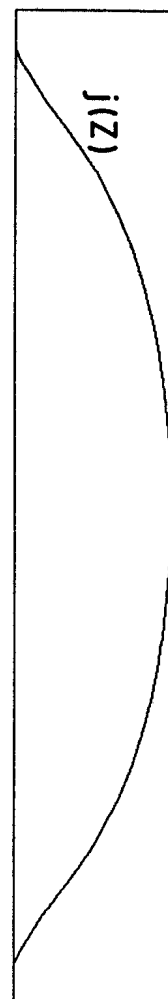
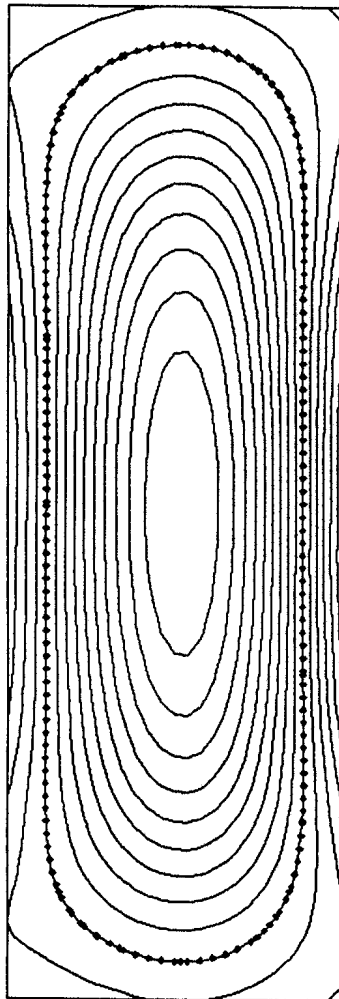
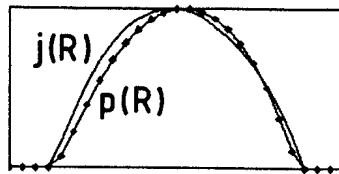
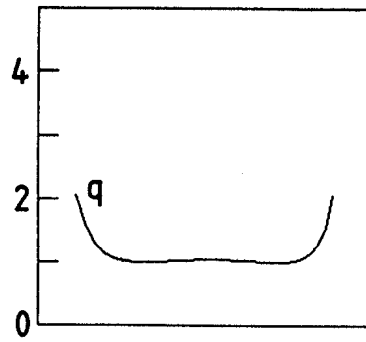


Fig.2a.

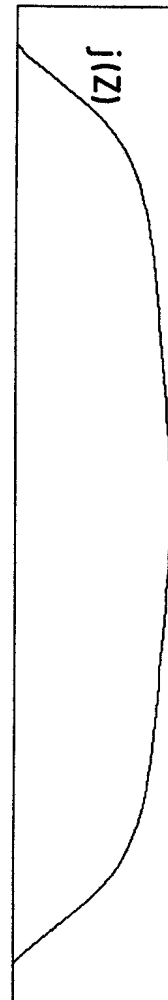
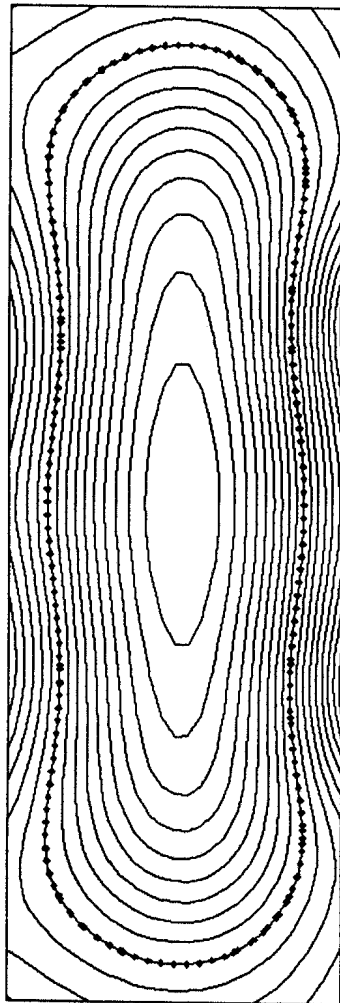
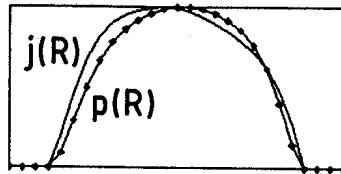
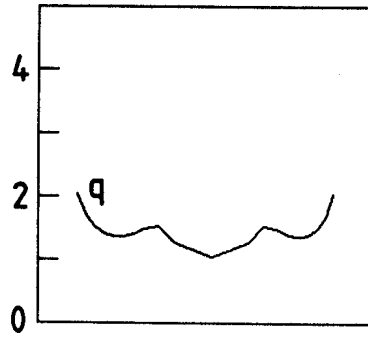


Fig.2b.

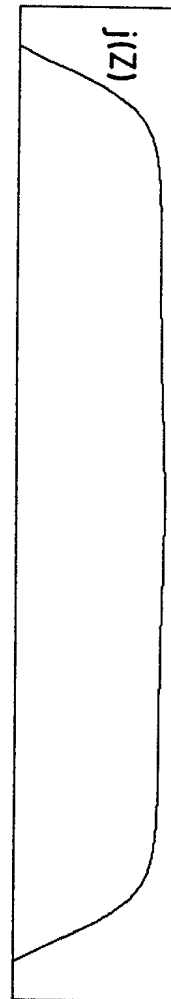
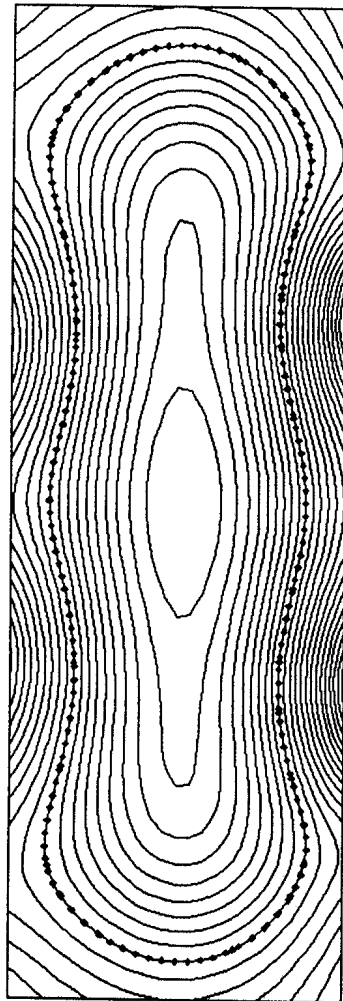
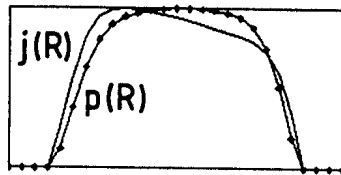
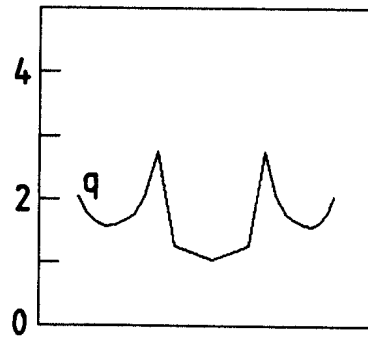


Fig.2c.

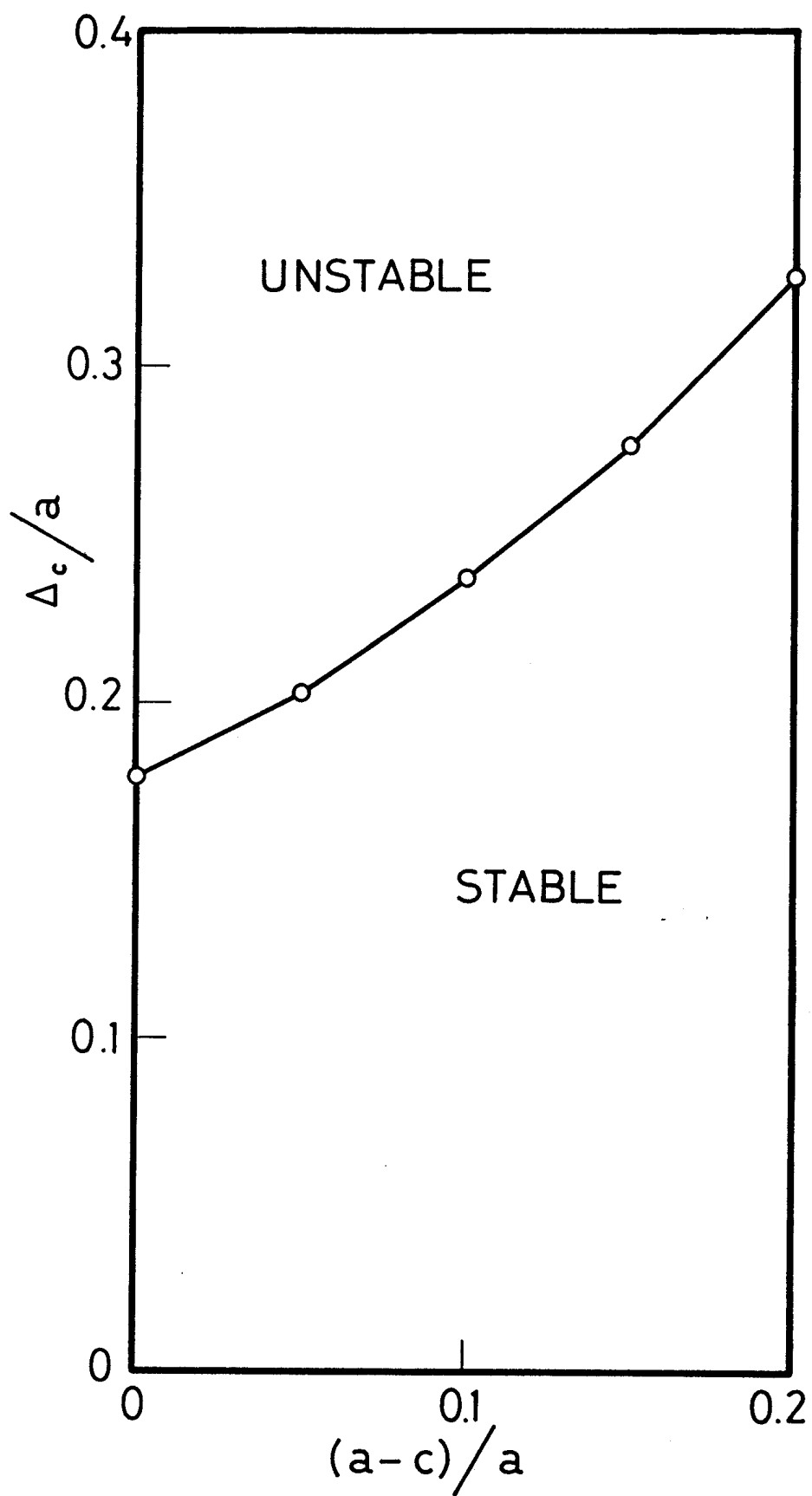


Fig.3.